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ABSTRACT

Fifty first- and second-grade children's understanding of logico-mathematical relations were assessed using selected Piagetian tasks to test four hypotheses. The children's abilities to conserve numbers were compared to the means of mathematics concept scores and total mathematics scores of the 1972 Stanford Achievement Test, Form A. The relationships between these Piagetian tasks and individual mathematics concept items on the SAT were assessed; and the relationship between children's stages of cognitive development and their mathematics class achievement ranking as measured by daily classwork were determined. The results were mixed with some Piagetian tasks showing a direct relationship with test scores and others showing none. (MP)

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SELECTED CONCRETE LOGICAL PIAGETIAN TASKS AS THEY RELATE TO MEAN AND ITEM SCORES ON THE MATH CONCEPT SECTION OF THE STANFORD MATH ACHIEVEMENT TEST, FORM A.

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A Graduate Research Project Submitted in Partial Fulfillment of the Degree of Master of Science in Education

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GENESEO, NEW YORK

CERTIFICATE OF APPROVAL

OF RESEARCH PROJECT

This is to certify that the Research Project titled, "Selected Concrete Logical Piagetian Tasks as They Relate to Mean and Item Scores on the Math Concept Section of the Stanford Math Achievement Test, Form A," which was presented to this department by Diane Marie Wolcott in partial fulfillment of the requirements for the degree of Master of Science in Education, has been approved as being of satisfactory quality.

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Selected Concrete Logical Piagetian Tasks as they Relate to Mean and Item Scores on the Math Concept Section of the Stanford Math Achievement Test, Form A

Children are given achievement tests to assess their ability to perform selected school tasks. While the purpose is often to determine how the school is performing, there is a need to determine the success of individual children; particularly to determine the skills they cannot perform and the concepts they do not understand. Such knowledge is essential in designing and prescribing remedial exercises for children who need them. Although achievement tests help in assessing children, they are likely to be graphic or symbolic in character and require of some children excessive skill with language. Thus, assessment can be augmented by the use of diagnostic tasks similar to those developed by Jean Piaget and his colleagues.

THE PROBLEM AND ITS SETTING

Statement of Problem

The purpose of this research project is to determine the relationship between the development of certain logical operations in first and second grade children as measured by selected Piagetian tasks and mathematical abilities assessed by a more traditional test of mathematical competency, the Stanford Achievement Test, Form A. The Piagetian tasks were: seriation, qualitative similarity, and ordinal correspondence; hierarchial classification and the inclusion operation; the provoked correspondence of number; the addition of classes; conservation and the measurement of length; and the arithemetical relations of parts to whole. The primary problem can be divided into four subproblems.

The Subproblems

1. The first subproblem. How do the means of scaled math concept scores and total math scores obtained by children on the Stanford Achievement Test, Form A, relate to the children's three stages of cognitive development in each of the selected Piagetian tasks?

2. The second subproblem. How do the means of scaled math concept scores and total math scores obtained by children on the Stanford Achievement Test, Form A, relate to the distribution of children performing at the first and third stages of cognitive development in each of the selected Piagetian tasks?

3. The third subproblem. Do significant differences exist between a child's stage of cognitive development on a particular Piagetian task and his ability to answer specific questions on the math concept section of the Stanford Achievement Test, Form A?

4. The fourth subproblem. Is a child's ability to conserve number as

measured by selected Piagetian tasks related to his math class achievement ranking as measured by his daily classwork?

The Hypotheses.

The first hypothesis is: the differences in the means of the math concept scores and in the means of the total math scores do not relate directly to the ability of the children to perform the selected Piagetian tasks at stages one, two, or three.

The second hypothesis is: the differences in the means of the math concept scores and in the means of the total math scores do not relate directly to the ability of the children to perform the selected Piagetian tasks at stages one or three.

The third hypothesis is: there is no significant association between children's stages of cognitive development on a particular Piagetian task and their ability to answer specific questions on the math concept section of the Stanford Achievement Test, Form A.

The fourth hypothesis is: there is no significant relationship between children's ability to conserve number as measured by selected Piagetian tasks and their math class achievement ranking as measured by daily classwork.

The Delimitations

There is no intent here to either rate the math teaching of the teachers of the children in the study or to predict the future success of the pupils involved in the study. This study is limited to first and second grade children who took the Stanford Achievement Test as a measure of accumulated mathematical knowledge.

Assumptions

1. It is assumed that Piaget's theory of cognitive development is operational among the subjects; they will gradually mature...developing meanwhile the ability to perform each of the tasks on a concrete operational level;

2. It is further assumed that Piaget's method clinique (explained in the section on methods, pp.26-29) can be used to assess the concrete operational ability of first and second grade children to use the selected number tasks. If these assessed abilities relate directly to mean scores of the Stanford Achievement Test (SAT), Form A, or certain item scores one might assume that either the test or the certain items on the test can be utilized in adequate assessment of the abilities of children to use the selected concrete number tasks. Remedial experiences could then be prescribed. If they do not relate directly, no such assumption can be made.

3. A child's development of his real intelligence is related to an age-stage sequence and that the child's thought processes differ from an adult's in the quantity and quality of available concepts;

4. The fourth assumption is that action oriented alternatives to the present standardized pencil and paper achievement tests need to be developed in order to eliminate their identified weaknesses; the effects of the need for children to use graphic and symbolic interpretations during assessments of their mathematical abilities and understandings of concepts.

OPERATIONAL DEFINITIONS

Accommodation The process of reaching out and adjusting to new and changing conditions in the environment; existing behavior is modified to cope with new information or situations.

Adaptation: A mode of functioning characterizing all forms and levels of life; consists of dual processes of assimilation and accommodation, which go on continuously.

Assimilation: The process of taking in from the environment all types of stimulation and information, which are then digested and reintegrated into the organism's existing structures.

Cognition: Mental or intellectual activity, such as, thinking, knowing, remembering, perceiving, recognizing, or generalizing.

Concrete Operations. Concerned with working logically with existing objects and includes ordering, seriation, classification, and mathematical processes.

Conservation. The invariance of the number of the set when the objects are rearranged.

Equilibrium. The process of regualting assimilation and accommodation to maintain a state of internal balance.

Experience. Refers to the physical and empirical experiences of children which contributes to their understanding of the world through their senses and their muscles.

Formal Operations. Deals with logical ideas and hypothetical reasoning, based on theoretical constructs rather that concrete objects.

Genetic Epistemology. The developmental study of the nature of knowledge, how it begins and how it continues.

Groupings. Organized reversible and logical thought structures in the sense that every element is related to the others. Characteristic of the stage of concrete operations.

INRC. Logico-mathematical groups characterized by four types of transformations: Identity, Negation, Reciprocity, and Correlation. Characteristic of the period of formal operations.

Intuitive Reasoning. Based on immediate perception; by trial-and-error may lead to correct conclusions.

Maturation. The development of patterns which are innate within the person and appear in sequential order in all normal development.

Operations. The interiorized activities of the mind; opposite to the sensory-motor or physical activities of the body. Characterized by

logical thought processes which are reversible.

Perception. The direct and immediate first impression of objects, people, or situations perceived.

Perceptual Reasoning. Based on immediate appearances; characteristic of the preoperational child who focuses on only one aspect of the environment or a stimulus at a time.

Preconcepts. The child's initial, hazy attempts at generalization; may confuse subsets of a class with the whole class.

Preoperational. The time period of early childhood preceding the development of logical, reversible operations. Characterized by egocentric thinking and illogical intuitions based upon perception.

Representation. The manner by which an image, a sign, or a symbol comes to represent an external reality. Memories are interiorized images, while words are verbal signs used to represent complexes of socially recognized meanings.

Reversibility. A characteristic of logical operations which allows the mind to reverse its activity and return in thought in order to coordinate previously recognized phenomena with current knowledge.

Schema. A mental structure or behavior pattern developing from the intergration of simpler, more primitive units into an enlarged and more complex whole.

Sensory-Motor. Refers to learning based on information received through physical exploration and sensory stimulation.

Social Transmission. A type of learning dependent upon verbal instruction and experiences of a social or cultural nature.

Stages. Levels of development characterized by successively more complex and more highly integrated patterns of thought or behavior. Usually associated with certain chronological ages.

Topology. The study of spatial relationships and forms without size or shape. Deals with open and closed figures, such as, a simple closed figure which divides a plane into inside and outside.

REASONS SUPPORTING THIS STUDY

Educators have utilized some form of evaluative measurement for many centuries. One of the earliest registered uses of formal evaluation was in the year 1115 B.C., at the inception of the Chan Dynasty in China, when examinations were constructed to select candidates for public office (Athey, p. 328). From these ancient Chinese tests to the present-day standardized multiple-choice exams, testing has been used to determine the degree to which people have cultivated particular types of ability or knowledge.

Due to the need to evaluate growing numbers of students in the Twentieth Century American school population, there has been an increasing dependence on standardized tests for evaluation. This has resulted in refinements in methods of evaluation and test construction which, in our free enterprize system, has evolved into a commercial service venture by experts not involved in teaching or school administration. Commercial psychological and achievement tests are used for a multitude of educational purposes throughout this country. This specialization has its weaknesses: the primary one being the separation of those who construct tests from those who teach and learn. This segregation has frequently resulted in misunderstanding of the underlying purposes of tests used to measure classroom achievement by school personnel, as well as misunderstanding of the aims of public schooling by those who construct the evaluation instruments. Achievement tests claim to measure a student's knowledge about selected facts and ideas that those who construct the tests believe to be important. Whether they measure what is or should be taught or learned remains a question. Furthermore, the verbal bias of most achievement tests may stand in the way of assessing a pupil's comprehenesion of some things he has been taught and understands.

Such standardized norm-referenced tests usually contrast the ability of one student with the ability of a predetermined group of pupils on a designated task. One's score is then denoted in terms of how he compares with the group. The score (be it scaled, percentile, grade equivalent, or stanine) is deemed a measure of relative achievement.

The belief in psychometric intelligence and achievement testing is so thoroughly implanted in American education and psychology that few school systems or professional journals can exact respect without considering IQ and achievement test scores in evaluating educational programs, in assessing students' progress, or in writing proposals for Federal or State grants. McClelland (1973), however challenged the reliability and validity of IQ tests as measures of intelligence. In her review of the literature relating to IQ testing, McClelland concluded that the high correlations between IQ scores and various evaluations of student achivement or adjustment are more apt to be due to social class rather than to innate intelligence.

The National Education Association, the American Federation of Teachers, and parent organizations have recently challenged the validity and limits of standardized school achievement tests. A review of the research dealing with achievement tests revealed numerous criticisms of them. The following suggestions of improving the structure and uses of these tests were made:

1) ceasing the practice of comparing children with one another through norm-referenced tests and substituting criterion-referenced tests that determine the child's ability to deal effectively with necessary tasks;

2) establishing the validity of test items in clinical settings in which the reactions of children from different ethnic and cultural backgrounds are noted;

3) demonstrating that performance on the test coincides with skills

needed in solving everyday problems and skills, such as: seriating, classifying, measuring, estimating, describing and solving word problems;

4) establishing means for providing rapid feedback that teachers can use in improving the learning environment of children;

5) redesigning the contents so they are more beneficial in helping teachers to determine the level of cognitive development of a child, and, in turn, to counsel and plan instruction that relates to the child's skills and capacities.

In the early school year, each of these suggestions can be related to the assessment of a child's ability to use a developing series of concrete-logical number tasks following tentative sequences first described by Jean Piaget. Athey (1970) and Greeen, Ford, and Flaner (1971) discuss the potential use of scales based on Piagetian tasks as a primary alternative to current standardized tests. There seems to be a number of strong advantages. In the first place, unlike conventional test questions, the Piagetian tasks have significance and meaning in a theory of developmental intelligence, instead of being empirically derived from a large number of questions which are distinctive of specific grade or age levels. Second, the reliance on reading ability or verbal comprehension is reduced; and, inferences about operational behavior can be made more directly. Being diagnostic in nature, the test materials may also be utilized to guide prescriptions for instruction, thus, a connection exists between the evaluation of intellectual achievement and classroom teaching. Third, students' performance on the Piagetian tasks is related to chronological age in simple and reliable developmental curves, and to date seems to be largely unaffected by socio-economic differences within Western cultures. Fourth, they produce scorable and scalable answers.

Like Darwin and Freud, Piaget organized his empirical data by

disclosing structures where others merely saw unrelated events or data. Einstein commented on Piaget's developmental theory as "the idea of a genious, such simplicity" (Furth and Hans, 1969, p. 6). Contrast Piaget's simple tasks with the questions of standardized tests that survive an extensive screening process. The items of most current test deal primarily with the content rather than the functioning of the intellect. These current evaluation techniques assume that a child improves in his ability to do such things as: to define terms, to recall facts, or to perform routine mathematical calculations from pre-kindergarten onward. They do not assume that different skills of using logic underscore the changing styles of cognitive behavior during the life of a child.

Piaget type tasks, in contrast, can be used to measure a pupil's competence in using different logico-mathematical skills at different levels of competency as he or she matures. These skills are often crucial in solving everyday problems as well as understanding math, science, and language. Selected Piagetian tasks can be used to assess the levels at which children comprehend and use selected logico-mathematical skills without worrying about overall labeling of children as mentally retarded or learning disabled.

Hood (1962) examined the development of number in normal, educationally subnormal who required some form of specialized education, and mentally defective English children. After reviewing Piaget's clinical method and several of his tasks he states:

> A teacher relying on chronological age grouping and making due allowance for exceptional cases, could feel reasonably sure that a large majority of under-fives would have little appreciation of pre-number concepts, that those above seven could usually be counted on to approach full understanding, but that in the two year period from 5 to 7 no assumption should be made (p. 276).

In 1955, Carpenter, as the result of research, suggested that the testing of retarded children demonstrates that prenumber concepts tend to mature

as general mental capacity increases. Hood's work with retarded children tended to support this thesis.

Even though many weaknesses of achievement tests can be identified, it is unlikely that those behind the traditional testing movement will heed all the criticisms or that the schools will observe the moratorium suggested by the NEA in 1976. What then can be done to improve the evaluation of children in schools? What does the literature suggest? Kohlberg and Mayer (1973) proposed that Piagetian "cognitive stage measures provide a rational standard for educational intervention where psychometric intelligence tests do not" (p. 489). This recommendation is in agreement with McClelland's (1973, p. 8) defense of tests with scores which fluctuate "as the person grows in experience, wisdom, and ability to perform effectively on various tasks that life presents to him".

The most judicious answer to the first question seems to involve improving standardized tests, to use those which are suitable, and to search for new and more appropriate approaches to assessing the abilities of children. Such improved tests will be more beneficial for evaluating student progress, for planning for instruction, and for forming educational decisions. On a long term basis, it now seems obvious that schools should hold test publishers responsible for producing tests based partially on the developmental levels of children, rather than only on traditional requirements of the curriculum and adults' ideas. Piaget and his colleagues recognize the child as a unique individual and not merely a miniature adult. Revitalizing the old model of the achievement tests as the commercial establishments and media seem to be calling for today is not enough. It is essential to furthering the education of children that we begin to assess children individually and prescribe educational activities on the basis of a developmental model. Educators might become actively involved in creating alternatives or complements to the current evaluation tests.

THE REVIEW OF THE RELATED LITERATURE

The issue concerning school achievement is: Do theoretical and empirical differences exist regarding the development of cognitive abilities as defined by Piaget and other mental abilities, as measured in classroom and standardized achievement tests?

The review of previous research related to this study will be divided into four subcategories: 1) Piagetian tasks and IQ or general achievement tests; 2) Piagetian conservation tasks and pupils' mathematical achievement; 3) Piagetian number tasks and mathematical achievement; and 4) Piagetian tasks and individual items on standardized math achievement tests.

Piaget's understanding of knowledge is more diverse than that reflected by school achievement tests. He views the development of knowledge and the unfolding of intelligence as the same; this contrasts with the psychometric sense of forming correct answers to particular test items. Piaget, in Ripple & Rockcastle (1964), demonstrated that specific learning is dependent on general cognitive development. Since it is the positive relationship between intelligence tests and achievement tests that lends credence and justification to the controversial use of these tests in the schools, the question arises as to whether Piagetian and achivement measurements coincide.

School achievement, according to Kohlberg and Mayer's (1973) examination of the literature, does not presurmise the child's ability to reason logically, which Piagetian tasks measure. Perhaps this is because his tasks deal with everyday experiences in which children utilize the processed information these tasks require, such as: number, classification, and quantity. Empirical relations are examined among the various Piagetian and psychometric assessments of intelligence and school

achievement by Rheta DeVries in Relationships among Piagetian, IQ, and Achievement Assessments (1974). The results of her investigations indicate that little overlap exists between knowledge on Piagetian tasks and school achievement cognition as evaluated by the Metropolitan Achievement Test. The correlation of merely .20 between number conservation and math achievement is especially significant. Factor analysis reveals that Piagetian, IQ, and achievement measurements coincide to some degree, but at the same time appraise diverse aspects of mental activity. Previous research with children by Almy (1970) and Kohlberg & DeVries (1969) also demonstrated that psychometric and/or achievement tests of mental development consider a different aspect of intellect than Piaget-inspired tests.

Hathaway's (1973) factor-analytic study of data gathered by Dudek, Lester, and Goldberg, identified three independent factors among 100 youths, 5-8 years of age: 1) WISC subtests; 2) seven Piagetian tasks; 3) California Achievement Test subtests, some WISC subtests, and four Piagetian tasks. Overholt (1965) showed IQ formed a more positive relation with conservers than non-conservers which is in contrast to Hathaway's independent factor of WISC subtests.

The general picture presented by these studies is unclear: some overlap exists among Piagetian tasks, IQ, and/or achievement tests, yet, theoretical differences also exist, appearing to correspond to real differences in cognitive evaluations.

Piaget's observations with students of various ages demonstrates some of the components in the enlarging competency to work systematically. The tasks, whether they deal with number and quantity, seriation, the logic of classification, or measurement of length, all reflect the degree to which the child is able to conserve, and the faculty he has in mentally reviewing his own cognitive steps. They are intended to allow the observer,

as long as he is well versed in Piaget's theroy, to portray the child's cognitive activity in regards to the logical operations involved.

A child's ability to conserve number is a prerequisite to understanding mathematical concepts and to solving mathematical word problems. If a child does not recognize the invariance of the number of a set when the objects are rearranged, he may have difficulty in comprehending mathematical relations such as addition, subtraction, multiplication, division, and fractions. The conservation of length presurmises a child's ability to compare the lengths' of objects. By determining if a relationship exists between a child's stage of development on Piagetian conservation tasks and his or her mathematical achievement, educators might be able to design compensatory experiences necessary for the "non-operational" child's formulation of mathematical concepts.

Williams (1958), in an analysis of the results of administering a number concept test to twenty children, found a high positive correlation between scores on a conservation task and the pupils' comprehension of the reversible relationship between addition and subtraction. However, he did not report the precise correlation and the sample size was limited.

A longituidinal study of the relation between conservation of quantity and school achievement was conducted by Almy (1966). Sixty -five students were tested. The correlations between conservation of quantity scores and a mathematics achievement test were .41 and .38 for the lower-class school, and .26 and .53 for the middle-class school. The results may have been influenced by two variables: the students in the middle-class school scored at the higher levels of the achievement test due to the easiness of the test for the group, and language was a difficulty for the pupils of the lower class school. If these factors did indeed influence the validity of the correlations, this would lend support to the practical

need for creating a mathematics achievement test that is not highly skewed by language development.

Grayson Wheatley's research (1968) points to the significance of Piaget's conservation of number concept as a predictive index of mathematical achievement. Of the three factors of the Number Concept Test (NCT): conservation, counting and cardination, conservation was the best single predictor of achievement. The correlations between the NCT designed for this project and the math section of the Stanford Achievement Test (SAT) was .75, significant at .001. The correlations of NCT with the four subsections of the SAT: concepts, computations, problem solving, and measurement were .59 except for the NCT with measurement, which was .76, indicating the important role conservation plays with measurement. This data suggests the NCT and the SAT ascertained, to a degree, similar factors. Conservers as a group scored significantly higher on the achievement test than nonconservers. Wheatley suggested further research to determine if conservation-style tests and intelligence tests supply diverse information.

The assumption is made that some skills taught in school, especially those requiring abstract reasoning, can not be developed until the appropriate stage of logical operations has been reached. Piagetian number tasks measure the child's logico-mathematical knowledge and/or reasoning ability. These tasks might be utilized to measure one's mathematical achievement. Although the assumption is reasonable, the degree of the dependence of school achievement on previous concept development has not been explained sufficiently.

Several researchers, among them Dodwell (1961), Hood (1962), and Freyberg (1966), have suggested the need for further study to eliminate this gap. Dodwell developed a standardized test of number concepts: correspondence, seriation, conservation, cardination and ordination, and

administered it to entering first-grade pupils. Although he considered the correlation of .59 between this test and a teacher-made test administered seven months later promising, he questioned the validity and reliablity of the teacher-designed test.

Hood (1962) found a sound degree of correspondence between level of cognitive development on devised Piagetian tests and teacher ratings of the children's arithmetical comprehension. The majority of those successful on the Piagetian tests made normal progress in number work; while there was "no instance of success in arithmetic and failure in the Piaget tests being found in the same child" (p. 284).

Relationships between general mental abilities, conceptual formation, and attainment in arithmetical computation and problem solving was explored by Freyberg (1966). A 72-question objective test measuring conceptual development showed a .523 correlation with the Primary Mental Abilities Test. Mental age and concept development scores indicated a moderate degree of association. DeVries's study (1974) also demonstrated that mental age was more closely associated with Piagetian measures than IQ scores. However, students at a low stage of conceptual formation did not demonstrate high arithmetic ability regardless of their mental age which is in agreement with Hood's findings (1962).

A study by E. Ross, reported in Hathaway (1973), identified four Piagetian classification tasks to establish a factor distinct from competence in classification as based on teachers' judgements and three Piagetian levels on number-related tasks. Ross found that no student on the lowest level on one measure was at the highest level on the other. Between these extremes, however, no linear relationship existed.

Freyberg suggested future research should attempt to determine "whether faster concept development contributes to higher school attainment or is a

reflection of it, or, in fact, whether both are the observed result of the operation of some third factor (1969, p. 159). The answers to these questions provide one of the keys to future improvements of teaching programs.

As was mentioned in a previous section of this paper, improvements are necessary in the construction of standardized achievement tests if we continue to use them to evaluate a child's understanding of mathematical concepts. The development of test items based on Piagetian number tasks could serve as a starting point. By using test items based on Piaget's theory to assess a child's understanding of mathematical concepts and relations, teachers can begin to make the math fit the child's development rather than forcing the child to fit the mathematics. Piaget's theory provides strong support and direction for formulating an effective evaluation method and for creating a beneficial match of content and student. At the present time, research demonstrating possible associations between Piagetian tasks and individual test items is lacking, providing support for putting the fourth hypothesis of this study to the test.

Three themes supporting the need for this study continue to emerge: 1) practicality; 2) an existing gap in the present research; 3) logical reasoning. Let us take a look at each of these themes.

Practical reasons. The use of Piagetian tasks to assess the logico-mathematical ability pupils' possess is time consuming. A quicker, related method of assessing similar abilities would be helpful to teachers, students, and those who construct achievement tests. Scales based on Piaget's tasks are supported by a theoretical conception of cognitive growth, rather than being empirically obtained from a large number of items which differentiate at specific grade levels.

Gap in the research. Few studies concerning the correlation between number conservation and classrrom mathematical abilities exist. Research

identifying specific achievement test items to selected Piagetian tasks is needed.

Logical reasons. By determining which achievement test questions measure a child's concept of number as selected Piagetian tasks do, educators might be able to use particular questions as screening devices to ascertain pupils' cognitive development and determine his readiness for the more advanced experiences needed to progress further. Overholt (1966) suggested that educators need more than teacher-designed and standardized conventional tests to evaluate student progress and understanding; providing support for the use of Piagetian tasks as evaluative instruments. By planning learning experiences in accord with the pupils' stages of cognitive development, the teacher will be presenting concepts in a manner such that pupils will learn to understand them and utilize them in solving mathematical or scientific problems.

Each of these reasons: practical, gap in the literature, logical imply a need to investigate more extensively the qualitative cognitive developments measured by Piaget-type tasks, their unique advantages in measuring individual intellectual development, and their educational implications.

RESEARCH DESIGN AND METHODOLOGY

Population

In order to test the four identified hypotheses, the sampling for this project involved fifty children, ranging in age from 6.48 to 8.72 years of age. All youngsters were enrolled in the first or second grade class in the Warsaw School System in Warsaw, New York, a small town in Western New York. The students attending this school are generally drawn from low or middle-class homes. Twenty-five children were tested from each grade. The enrollment for the 1976-1977 school year in grade one was 107 students; grade two, 76 students; and the total for kindergarten to sixth grade was 652 students.

Forty students representing the ten highest and the ten lowest students in math class achievement, referring to the students' understanding and ablility in daily math lessons as measured by assignments and teacher-made tests for each grade, were selected by the team leader after conferring with associates. The remaining five per grade were randomly selected from the class roster.

Each child was familiar with the investigator since she visited each classroom preceding the examination period. Additional time was spent informally with each pupil prior to the examiner's presentation of the selected conservation tasks.

The data was compiled from April 29 to June 10, 1977. A child was given no indication as to whether his reply was accurate or inaccurate. The investigator gave no value assessment on the subject's responses. No feedback on "right" or "wrong" performance was provided. Each task was presented to the subject as an exploratory activity with ample opportunity

to manipulate the diverse materials. Each session took about thirty minutes, after which the child returned to his regular classroom activities.

Administrators and teachers were familiar with the general structure of the project, but were not cognizant of the specific cognitive tasks to which the children were exposed. This precluded any endeavor on the teachers part to expose pupils to conservation problems ahead of time, since according to Piaget's theory, if a child lacks a particular concept it cannot be taught to him directly. Rather, possible teaching or training leads to pseudo concepts that are challenged when the materials or queries representing the concepts are altered. The understanding that the child must develop the concept from his own experiences suggests that direct instruction is of little value.

The Standardized Achievement Test

The 1972 edition of the Stanford Achievement Test, (SAT), Primary Levels I and II, Form A, was selected as the test most suitable for the curriculum in the school district being used in this study. The district administrators made the choice. The arithmetic section consists of mathematics concepts, computations, and applications; the latter two components combined as one unit for the first-grade students, individual units for second-grade students.

The SAT was administered to the subjects in a group setting in their homerooms, May 22-27, 1977.

Statistical Approaches

Analysis of Variance was used to find out if the means of math concept scores or total math scores on the SAT (Form A) differed significantly from children's level of cognitive development, for 1) stage one, stage two, and stage three; or 2) stages one and three. Note that there is one criterion of classification for each hypothesis: stage of cognitive development measured by the selected Piagetian tasks.

The first step in an ANOVA is the breakdown of the total variance (σ^2) of scores into two classes: 1) the variance related to the different conditions being the three stages of cognitive development for each task, or the variance among the means, and 2) the variance developing from individual differences within the groups or conditions. One may then ascertain if the group means differ significantly per se in terms of the variability within the separate groups, that is, individual differences, by dividing the "among means" variance by the "within groups" variance and comparing the resulting variance ratio, called the F ratio, with the values in a statistical F table. In order to reject a null hypothesis, the ratio of two variances is examined to see if the deviation from 1.0 is sufficiently large.

The total number of degrees of freedom for each ANOVA is equal to the number of students involved in the task minus one. The F value to be equaled or exceeded depends upon the degrees of freedom (df). The df for "among the means of conditions" are equal to the number of conditions minus one. The df "within groups" or "within conditions" are equal to the difference in the total df and the df for among the means of conditions.

If N= number of total scores and k= number of conditions or groups, the general case is:

df for total = (N-1)
df for among means of groups = (K-1)
df for within groups = N-K

The chi square (χ^2) test is utilized to appraise whether there is a significant difference between expected frequencies and the corresponding observed frequencies in at least two categories with at least two groups. It is a nonparametric statistical methodology. It makes no assertions about parameters nor about the form of the distribution composing the data.

The two-way chi square test was used in this study to test the third and fourth hypotheses. The differences between the observed and calculated expected frequencies are determined, the differences are squared, the squared difference is divided by its expected value, and the results of each of the cells are summed.

The set-up for hypotheses 3 and 4 is shown in tables 1 and 2; df= (3-1) (2-1) =2. Only two cell frequencies are allowed to vary; the remaining frequencies are stabilized. The computed chi square value is compared with the critical value, $_{.05}\chi^2$ df, and $_{.01}\chi^2$ df, located in a chi square table. The significance or non-significance of the chi square-value is then determined. In addition to demonstrating a significant deviation between observed and expected cell frequences, a significant χ^2 can be recognized as either a significant difference between levels of one of the variables or as a significant relationship between the two or more variables. The null hypothesis is accepted or rejected as appropriate.

Table 1 is an example of the 3 x 2 table used to test the third hypothesis: There is no significant association between children's stages of cognitive development on a particular Piagetian task and their ability to answer specific questions on the math concept section of the Stanford Achievement Test, Form A. An individual table was developed for each identified test question appearing to relate to a Piagetian task.

Table 1

Developmental Differences in Accuracy of Responses of Children for SAT Questions

		Type of Response		
		Correct	Incorrect	Totals
Developmental Stage for Piagetian Task	I	Cell # 1	Cell # 2	
	II	Cell # 3	Cell # 4	
	III	Cell # 5	Cell # 6	
	Totals			N = 25

Using logico-mathematical knowledge, the investigator determined the Piagetian task most closely related to an individual test item. The observed frequency for each cell was obtained by tallying the number of children per stage of development for the specific Piagetian task and their accuracy in responding to the test item. The expected frequency was obtained for each cell by dividing the total number of students by the observed frequency. These two values were than compared by the chi sqare test.

The fourth hypothesis: there is no significant relationship between children's ability to conserve number as measured by selected Piagetian tasks and their math class achievement ranking as measured by daily classwork was tested by charts similar to table 2 for each of the six selected Piagetian tasks. Only those students ranking as either being in the top 10 or bottom 10 of their first or second grade class in terms of daily math achievement were compared to their stages of cognitive development through the chi square test. Thus, a total of only 40 subjects were utilized for testing the fourth hypothesis.

Table 2

Developmental Differences and Math Class Achievement Ranking

		Math Class Achievement Ranking		
		Top 20 Subjects	Bottom 20 Subjects	Totals
Developmental Stage for Piagetian Task	I	Cell # 1	Cell # 2	
	II	Cell # 3	Cell # 4	
	III	Cell # 5	Cell # 6	
	Totals			N = 40

Method Clinique

Piaget's study of his own children'a mental development led to the formation of his method clinique, now referred to as the method of critical exploration, which combines natural observation of students with the observation of their behavior in specially designated situations. This method is concisely described by Clapararède as follows:

"The clinical method is the art of questioning: it does not confine itself to superficial observation, but aims at capturing what is hidden behind the immediate appearance of things. It analyzes down to its ultimate constituents the least little remark made by the child. It does not give up the struggle when the child gives incomprehensible answers, but only follows closer in chase of the ever-receding thought, drives it from cover, pursues, and tracks it down" (Piaget, 1926, Introduction).

The experimental situations used by Piaget involve various types of material and involve the students in formulating judgements about numbers or quantities, and usually requires manipulation of the materials. A description of his tasks may be found in The Child's Conception of Number (1952a) and will not be reiterated here.

This project was designed to test each student on these six cognitive tasks: 1. Seriation, Qualitative Similarity, and Ordinal Correspondence; 2. Hierarchial Clsssification and the Inclusion Operation; 3. Provoked Correspondence of Number; 4. Addition of Classes; 5. Conservation and the Measurement of length; and 6. Arithmetical Relations of Parts to Whole. These tasks were adopted with little alteration from Piaget's experiments as described in The Child's Conception of Number (1952a). The tasks were verbally structured, requiring comprehension of the terms used and the experimenter's actions, manipulation of materials

generic to the task, and responding verbally to the investigator's queries. The tasks' questions were modeled after Piaget's original research and replication studies (Piaget, 1952a, 1969b,; Dodwell, 1961). The questions sought to ascertain the child's comprehension of equivalency and reversibility as involved in number and length relationships by determining the child's stage of development for each task.

Each individual session was recorded for later analysis. Descriptions of the tasks follow which include each one's purpose, materials, and the basic questions asked.

Task 1: Seriation, Qualitative Similarity, and Ordinal Correspondence.

Purpose: Number may refer to cardinality of objects in a set or ordinality, the position of an object in a set. Piaget contends that the child's understanding of ordinal number develops in close relation with his cardinal number concepts. This task assesses when children can arrange a set of objects in order by a size relation. When can children match two groups of objects in a specific order? Finally, when can children translate the idea of the position inhabitated by the objects into the concept of ordinal number?

Materials: Paper cut-outs: 1. Ten dolls of graduated size, the first, half the size of the last. 2. Ten canes of graduated size, but not quite as much variation in length as the dolls.

Procedure: The investigator placed the dolls and the canes on the table and requested the child to arrange the dolls and canes so it will be easy for each doll to find its cane to take a walk. The size of the dolls and canes and their order was noted. 2. After the child had seriated the dolls and canes by size, one row below the other, the row of dolls was moved closer together as the child watched. He was asked, "Which stick belongs to the smallest doll; the second largest; the fourth from the smallest? Why?" The dolls were not mentioned, but pointed to as the investigator asked the questions. 3. Reverse the order of one of the rows. Point to a doll (third) and ask, "Which cane belongs to this doll? Why?" 4. Disarrange the order of the dolls and canes and request the child to find the correct size cane for a particular doll. "Why did you choose that cane?" 5. Intermingle both series and select one doll. Instruct the child to: "find all the dolls larger than this one who will accompany the doll on a walk. Find the sticks for the dolls going on the walk. "Why did you choose these dolls; these sticks?"

Task 2: Hierarchial Classification and the Inclusion Operation

Purpose: In order to deal with logical classification a reversibility of thought is necessary for the child to simultaneously consider the whole and its parts. This activity is useful for identifying the difficulties of the child who recognizes classes (Stage II). but fails to comprehend the hierarchial relation between classes, that is, flowers include roses, and roses include yellow roses.

Materials: A set of twelve pictures which forms a hierarchial classification dealing with flowers, roses, and yellow roses, Each flower was drawn on a six by nine inch sheet of white construction paper.

Procedure: The pictures were placed before the child with the directions to "put them together so they are alike." General questions as to color or type of object helped to guide his thinking, such as, "If you made a bouquet out of all the flowers, would you use the yellow roses?" The child was then asked questions of four types.

Questions:

1. "Is a bunch of all the yellow roses bigger or smaller than a bunch made of all the roses? Why?"
2. "Are there more roses or more flowers? Why?"
3. "If you take all the roses, will there be any flowers left? Why?"
4. "If you take all the flowers, will there be any roses left? Why?"

Task 3: Provoked Correspondence of Number

Purpose: It is frequently assumed if a child can identify the number of objects in a collection, he comprehends the meaning of the number. This activity demonstrates that counting is not the main criterion by which to determine the child's understanding of numbers. Does the student still believe the number of objects is the same when the objects are spread apart? Does he recognize the conservation of number concept?

Materials: Ten artificial roses; seven vases.

Procedure: The student is presented with a row of vases and a pile of flowers nearby. Piaget suggests "the closer the one-to-one correspondence between elements, the more lasting the equivalence of the corresponding sets will be" (Piaget, 1952a, p.49). With the seven vases in a row and the pile of flowers nearby, the child was asked to take just enough flowers so each vase would have one flower. Extra flowers were removed from the table. He checked his work by placing a flower into each vase, therefore, establishing a one-to one correspondence, and a clear perceptual relation between the two sets.

Questions:

1. If a child manages to establish the one-to one correspondence, the flowers are removed and placed in a group on the table.. "Are there still the same number of flowers and vases or are there more in one group? "Why?"
2. Instruct the child to place the flowers beneath the vases to maintain the one-to-one correspondence. Spread the vases apart so they extend beyond the row of roses. "Are there still the same number of flowers and vases or are there more of one group--flowers or vases?" "Why?"

Task 4: Addition of Classes

Purpose: To determine when and how the growth of the child's simplest notions of logical relations fits in with that of his elementary arithmetical ideas. Thusly, one utilizes the purely logical relation which comes closest to a numerical one, that of parts to whole. Addition will be considered here as applied to classes of objects, in this instance, beads. A similar concept was determined in the class inclusion relation which considered the hierarchial relation between yellow roses, roses, and other flowers.

Materials: Twelve plastic beads that snap together; nine pink and three white ones.

Procedure: Place the assorted beads in front of the child and encourage the child to examine and/or play with them.

Questions:

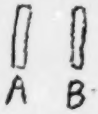
1. "What kind of material are the beads made of?"
2. "What color is this bead (pointing at a pink bead) and this bead (pointing at a white bead)?"
3. "Are there more pink beads or more plastic beads?" "Why?"
4. "If I made a bracelet of plastic beads and a bracelet of the pink beads, which one would be longer?" "Why?"

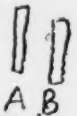
Task 5: Conservation and Measurement of Length

Purpose: Measuring the length of an object would be arduous if not impractical if the length of the ruler changed as it was moved along the object to be measured. Yet many students believe the length of an object is altered as you move it in relationship to a second object. Too often, his judgement is limited to perceptual factors rather than using the logic of identity which this task attempts to ascertain.

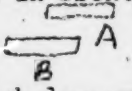
Materials: Two ice-cream sticks, one milimeter by eleven milimeters.

Procedure: The child is shown two ice-cream sticks as positioned:

Questions: 

1. "Are the sticks the same length or is one stick longer?" "Why?"
   If the child decides one stick is longer, place the sticks closer together until he is convinced that they are the same length.
2. Then move one stick toward yourself about twenty millimeters.
   "Are the sticks the same length or is one stick longer?" "Why?"
3. Move stick A ahead of stick B. 

   "Are the sticks the same length or is one stick longer?" "Why?"
4. Move stick B into a horizontal position.

   Are the sticks the same length or is one stick longer?" "Why?"
5. Move stick A to a horizontal position extended in front of stick B. 

   "Are the sticks the same length or is one stick longer?" "Why?"

Task 6: Addition and Subtraction of Numbers

Purpose: Combining a class of flowers with another class of flowers we still have a class of flowers, but in uniting a counting number with itself another number is the result. Psychologically then, is addition of numbers more difficult than addition of classes?

According to educators and psychologists, the ability to add numbers is found to form at the same time as the ability to add classes since the same reasoning processes of conservation of number and reversibility of thought are prerequisites. Does the student perceive the whole as being invariant in terms of distinctive additive combinations such as 4 + 4, 1 + 7, or 6 + 2?

Materials: Eight plain M & M candies, all the same color..

Procedure: The child is shown two groups of sweets arranged as follows: OOOO OOOO ----First day---- He is asked to listen carefully to this story: Today he may have four sweets at eleven o'clock and four sweets after school. Tomorrow he may have the same amount, but in different denominations--- one in the morning and seven in the afternoon.. At this point, remove three of the sweets from the set on the left and place them with the set on the right to represent the sweets for the second day.

O OOOOOOO Second Day

Questions:

1. "Will you have more sweets on one day or will you have the same number on both days?" "Why?"
2. To be sure the question is understood, both sets of the second day are pointed out and the question is asked, "Together, are these sweets the same amount as those on the first day?" "Why?"

When the questioning was completed, the child was given the M & M candies to eat.

Treatment of Data

Scoring of the student's responses determined whether he was identified as capable of solving a Piagetian task, and thusly, which stage he was in of cognitive development. In the traditional Piagetian problem, logical justification of the judgement "same" must be given for the child to be judged a conserver. Some studies, however, have not required this explanation.

Gruen suggests:

> It is an open question as to which set of criteria is the appropriate one for assessing the presence or absence of conservation, but it is not an arbitrary one. The choice depends to a great extent on the psychological processes that one assumes underlie conservation. For example, the criteria that Smedslund has used seems more appropriate for assessing conservation as Piaget defines it than do the criteria used by Frank Bruner, (1964). This is because Smedslund, like Piaget, believes that logical operations such as reversibility, compensation, and logical necessity underlie conservation and must be present in the thinking of a child who has conservation" (1966, 37, p.982).

Using logical adjudication seems to be the only way to truly test Piagetian theory. "The operational structure as defined by Piaget underlying the conservation concepts appear to us to be a complex, co-ordinated system that cannot be properly evaluated by rather summary investigation of answers to preselected questions with no exploration of the child's justification of those answers" (Inhelder, Smock, Bovet, and Sinclair, 1966, 21, p. 163).. In this research project, the investigator followed questions with a "Why?" or "How do you know that?" query. This helped to determine the reasoning used by the child to justify his judgements and, thereby, determine his stage of cognitive development. In fact, it is often these arguements that demonstrate the essence of the intrinsic barriers the child's reflection encounters.

The process of scoring is a complicated task even if several trials are given for each task as was done in these experiments, each of which was scored on the basis of logical reasoning and an overall score for each

task determined. The student's responses provided an assortment of replies, suggesting various logical types. These replies which have been found in Piagetian experiments formed the basis for the investigator's scoring individual answers and the overall score for a conservation task: Reversibility statements, "You can put the M & M candies back in one row and the number will be the same." "You can put the flowers back into the vases and you will still have the same number of each since there will be one flower for each vase." "You can move the stick back to where it was and it will be the same size as before, so it has to be the same size as the other stick."

Identity statements, "They are the same--you did not add or take any flowers or vases away." "It's the same length. You didn't make the stick longer or cut anything off the end."

Compensatory statements, "This stick (A) is longer at this end, but smaller at the other end, so the two sticks are still the same length." "Nothing has changed, you simply pushed the flowers closer together." "This time (second day) has a big group of seven and this group has one, but on the first day we had four here and four here so they are the same number of candies, but they just look different."

Tautology, or reference to a previous state, affirms the equivalency of members of a class. "A bunch of flowers is still a bunch of flowers, even if you add more mums. "The sticks' lengths are the same no matter where you move them; it only appears to make one of the sticks look longer than when we began."

Additive composition recognizes that parts can be put together in various ways without altering the total composition. For example, "The plastic bracelet is longer since it is made of both the pink and the white beads, while the other bracelet has only the pink beads." "There are more flowers

than mums since the flowers make up all the different kinds."

In many cases, only one of these forms of logical reasoning was used to justify responses to individual questions. The mention of only one of these responses for each complete task is an insufficient denotation of the presence of conservation of operative structures. Therefore, various situations were utilized for each concept which increased the probability of getting a more adequate demonstration of the child's abilities. The subject was encouraged to use various arguements so the investigator could determine their understanding in order to ascertain the extent of the subject's operatory structures.

Since each situation was taped, the experimenter was able to systematically review a subject's manipulations during the session, such as, the ordering of canes and dolls and his ability to handle the various tasks. The transcriptions and manipulations were then analyzed according to specific guidelines, described in part above, to determine the child's level of cognitive development. Piaget's research and replication studies suggest a classification system used for coding the data of this experiment (Dodwell, 1960; Elkind, 1964; and Piaget, 1952a, 1969b).

Reliability and Clinical Assessment

The usual problems regarding the evaluation of young children whose replies are extremely mixed are increased when the researchers attempt to combine Piagetian tasks and more traditional methods. The possibility of the "false positive" answer exists when the student is judged as thinking at the operational level when, actually, his response misleadingly suggests that degree of comprehension. Perhaps less probable, is the "false negative" when the child replies as though he did not comprehend the task or question since he has not recognized the purpose of the investigator soon enough and is not supplied extensive opportunities to substantiate his position.

Recognizing that the number of problems presented the child is sufficient to examine his representative manners of thinking about a specific task, what is the probability that he will answer in the same style if the question is reiterated? If the initial replies can, without a doubt, clearly be defined as operational, this stage should be maintained over subsequent interviews. If he is definitely pre-operational and the interviews take place within a brief time span, his initial replies should be sustained. Allowance should be made, however, for the feasibility of his primary rejoinders portraying impulsive action and that learning and development may intercede to alter them. The situation of the student who is designated transitional is even less predictable, since he may waver in his answers and subsequent explanations until the third stage is solidly confirmed.

All of the protocols were classified by this researcher. Prior to the present study, the investigator utilized Piagetian tasks to assess children's stages of development for a graduate course and used them successfully in a remedial math program in the Warsaw School System for diagnosing students' abilities to deal with various mathematical concepts. Since the interviews were recorded, the investigator was able to transcribe and code the children's responses following recommendations of previous researchers for the classification of replies.

Reporting of Results

The results of testing the four hypotheses will be reported in tables and discussed in the following section. Tables 5 and 6 correspond to the first hypothesis; tables 7 and 8 to the second hypothesis; tables 9 and 10 to the third hypothesis; and table 11 to the fourth hypothesis.

THE DATA

The primary data used to test the four hypotheses of this study are reported in tables 3 and 4, pages39 and 40, for first and second grade students, respectively. For each student, his or her stage of cognitive development is recorded along with the scaled math concept and total math score. The math class achievement ranking for the students is identified as "T" for top 10 of class, "B" for bottom 10 of class, and "M" for students who were chosen at random from the students ranked between the highest and the lowest students in terms of math class achievement.

Hypothesis 1: The differences in the means of the math concept scores and in the means of the total math scores do not relate directly to the ability of the children to perform the selected Piagetian tasks at stages one, two, or three.

The data used in the ANOVA for this hypothesis involved the students' stages of cognitive development for individual tasks located in columns 2-7 as the conditions and either the students' math concepts scores or total math scores located in columns 8 and 9, respectively. The children's stages of development and math concept or total math scores were compared for each task: six ANOVA for math concept score, six ANOVA for total math score. This data is reported in tables 3 and 4 for first and second grades.

Hypothesis 2: the differences in the means of the math concept scores and in the means of the total math scores do not relate directly to the ability of the children to perform the selected Piagetian tasks at stages one or three.

To test this hypothesis, one uses only the students' determined as being in stage I or III of cognitive development for the particular Piagetian task which is located in columns 2-7 in tables 3 and 4. The math concept scores are located in column 8, the total math scores in

Table 3

First Grade Students Developmental Stages for Six Select Piagetian Tasks, SAT Math Concept Scores, Total Math Scores, and Math Class Rank

Student	Task 1	Task 2	Task 3	Task 4	Task 5	Task 6	Math Concepts Score	Total Math Score	Math* Class Rank
1	II	III	III	III	III	II	143	134	T
2	III	III	III	III	III	II	155	148	T
3	I	III	II	I	I	I	104	112	B
4	III	II	II	II	I	I	122	129	M
5	III	II	III	I	III	III	139	139	T
6	I	II	II	I	I	III	96	112	B
7	III	III	III	III	III	III	139	143	T
8	III	III	III	III	III	I	143	137	M
9	I	II	II	I	I	I	106	110	B
10	III	III	III	III	I	III	139	134	M
11	III	II	III	III	III	I	182	182	T
12	II	I	III	III	III	I	136	136	T
13	I	II	III	I	I	I	110	117	B
14	II	III	III	III	II	II	123	130	T
15	I	III	II	III	I	I	110	122	B
16	II	I	II	I	I	I	96	112	B
17	II	II	III	III	I	I	170	161	T
18	II	II	III	I	I	I	104	113	B
19	III	III	III	III	III	III	155	156	T
20	I	II	II	II	I	I	106	117	B
21	I	III	II	III	I	I	104	114	B
22	I	III	III	III	I	I	131	134	T
23	I	II	III	II	III	I	106	120	M
24	I	II	II	I	I	I	110	119	B
25	I	I	III	III	III	I	139	143	T

*B--Bottom 10 Math Class Ranking
M--Middle Math Class Ranking
T--Top 10 Math Class Ranking

Table 4

Second Grade Students Developmental Stages for Six Select Piagetian Tasks, SAT Math Concept Scores, Total Math Scores, and Math Class Rank

Student	Task 1	Task 2	Task 3	Task 4	Task 5	Task 6	Math Concepts Score	Total Math Score	Math* Class Rank
26	II	II	III	III	I	II	119	127	B
27	I	II	III	I	I	I	137	131	B
28	II	III	III	III	III	III	155	161	T
29	II	II	III	II	III	I	130	125	B
30	II	III	III	I	III	III	151	156	T
31	III	III	III	III	III	I	163	173	T
32	III	II	III	III	III	III	170	161	T
33	II	II	III	II	I	I	137	134	B
34	I	II	III	III	II	I	117	129	B
35	III	II	III	II	II	II	146	155	M
36	III	II	III	III	III	III	151	159	T
37	I	II	III	I	I	I	125	120	B
38	II	II	III	III	I	I	140	137	M
39	II	I	III	I	I	I	127	130	B
40	III	III	III	III	III	II	182	173	M
41	II	III	III	III	I	III	131	133	B
42	III	III	III	III	III	III	170	159	T
43	III	I	III	I	I	III	136	126	B
44	III	II	III	III	III	III	151	150	T
45	III	II	III	III	III	I	182	173	T
46	III	II	II	I	III	III	151	156	T
47	III	III	III	III	III	I	170	168	T
48	II	III	II	I	III	II	131	134	B
49	II	II	II	I	II	I	130	128	M
50	II	III	III	III	I	I	136	139	M

*B--Bottom 10 Math Class Ranking
M--Middle Math Class Ranking
T--Top 10 Math Class Ranking

column 9. The ANOVA test is performed to determine how the means of math concept or total math scores differ for children in stage I and III of mental development.

Hypothesis 3: there is no significant association between children's stages of cognitive development on a particular Piagetian task and their abililty to answer specific questions on the math concept section of the Stanford Achievement Test, Form A.

Individual items on the SAT Form A , were identified as to the selected Piagetian tasks which they might be attempting to measure. The analysis of individuals' answers to the math concept questions is available from the investigator. Pupils' levels of cognitive development for the six Piagetian tasks are found in columns 2-7 in tables 3 and 4.

Hypothesis 4: there is no significant relationship between children's ability to conserve number as measured by selected Piagetian tasks and their math class achievement ranking as measured by daily classwork.

To test this hypothesis, one compares the pupils' stage of cognitive development for the individual Piagetian task located in columns 2-7 and their math class ranking located in column 10 of tables 3 and 4. A chi square test was utilized to compare these two variables.

THE RESULTS

The results of examining the data necessary for testing the four hypotheses is reported and interpreted for each individual hypothesis in table and written form. The data utilized is described in tables 3 and 4. Each hypothesis will be stated; followed by the findings related to it.

First Hypothesis: The differences in the means of the math concept scores and in the means of the total math scores do not relate directly to the ability of the children to perform the selected Piagetian tasks at stages I, II, or III.

The math concept scores of the children were tabulated for the three stages of cognitive development for each of the six selected Piagetian tasks. The means of the scaled scores were determined by stages of development to ascertain their relationship. Examination of table 5 indicates that the means do vary significantly, $p < .01$ for Task1, Task 3, Task 4, and Task 5. For Task 1, the F ratio of 25.241 is highly significant, as is the F of 20.396 for Task 3. The F ratios 8.644 and 16.405 for Task 4 and Task 5, respectively, are greater than the critical F ratio of 5.11. The means of math concept scores of students in each stage are not equal, at least one of them varies for each task, therefore, the hypothesis is rejected.

For Task 2, 1.165 is the F ratio, $p > .01$. For Task 6, 2.502 is the F ratio, $p > .01$. These ratios fit comfortably with the critical F ratio. The means of math concept scores do not differ significantly in variability for the three stages of development, suggesting the level of achievement on the math concept test is independent of students' stages of intellectual development for the hierarchial classification and the inclusion operation task and for the arithmetical relations of parts to whole task.

Table 5

Hypothesis 1: The differences in the means of the math concept scores and in the means of the total math scores do not relate directly to the ability of the children to perform the selected Piagetian tasks at stages one, two, or three.

Relationship between Means of Math Concept Scores on the SAT, Form A

Source	df	Sum of Squares	Mean Squares	F Ratio
By Task 1: Seriation, Qualitative Similarity, and Ordinal Correspondence				
Total	49	26299.2800		
Between	2	13619.3536	6809.6768	25.241 [a] *
Within	47	12679.9264	269.7857	
By Task 2: Hierarchial Classification and the Inclusion Operation				
Total	49	26299.2800		
Between	2	1242.4900	621.2450	1.165 [a] **
Within	47	25056.7900	533.1232	
By Task 3: Provoked Correspondence of Number				
Total	49	26299.2800		
Between	1	7842.5607	7842.5607	20.396 [b] *
Within	48	18456.7193	384.5150	
By Task 4: Addition of Classes				
Total	49	26299.2800		
Between	2	7072.4139	3536.2070	8.644 [a] *
Within	47	19226.8661	409.0823	
By Task 5: Conservation and Measurement of Length				
Total	49	26299.2800		
Between	2	10811.8017	5405.9009	16.405 [a] *
Within	47	15487.4783	329.5208	
By Task 6: Arithmetical Relations of Parts to Whole				
Total	49	26299.2800		
Between	2	2530.3046	1265.1523	2.502 [a] **
Within	47	23768.9754	505.7229	

Note. [a] $F_{.01}(2, 45) = 5.11$ * $p < .01$

[b] $F_{.01}(1, 50) = 7.17$ ** $p > .01$

The total math scores of the students were tabulated for the three stages of cognitive development for each of the selected Piagetian tasks. Means of the scaled scores were measured and compared with the stages of development todetermine their relationship. The analysis of variance in Table 6 shows a significant degree of variance in the scaled scores' means and Task 1, Task 3, Task 4, and Task 5 at the $p < .01$ level of significance. The MS_B is significantly greater than the MS_W for each of these tasks producing an F ratio exceeding both the critical F ratios at $p < .01$ and $p < .05$. For Task 1, $F = 22.999$; for Task 3, $F = 14.324$; for Task 4, $F = 9.007$; for Task 5, $F = 18.622$. The hypothesis is rejected for these four tasks.

Any F below the critical ratio, 5.11, would lead one to recognize that means of total math scores for three stages do not differ significantly in variability. Thus, the F 1.44 $p > .01$ for Task 2, hierarchial classification, allows one to accept this hypothesis. The F 2.612, $p > .01$, for the arithmetical relation of parts to whole task does fit comfortably into the hypothesized chance distribution. This hypothesis is also accepted for Task 6.

The first hypothesis, claiming no significant relations between the means of math concept scores and total math scores on the SAT and the children's three stages of cognitive development is partially rejected: a significant difference exists between the variables for Task 1, Task 3, Task 4, and Task 5. This hypothesis is accepted for Task 2 and Task 6.

The scores of children in stage two, considered a transitional stage, were not considered in the analysis of variance of mean math concept scores and total math scores for each task in hypothesis two.

Second Hypothesis: The differences in the means of the math concept scores and in the means of the total math scores do not relate directly

Table 6

Hypothesis 1: The differences in the means of the math concept scores and in the means of the total math scores do not relate directly to the ability of the children to perform the selected Piagetian tasks at stages one, two, or three.

Relationship between Means of Total Math Scores on the SAT, Form A

Source	df	Sum of Squares	Mean Squares	F Ratio
By Task 1: Seriation, Qualitative Similarity, and Ordinal Correspondence				
Total	49	17712.0200		
Between	2	8760.6719	4380.3360	22.999 [a] *
Within	47	8951.3481	190.4542	
By Task 2: Hierarchial Classification and the Inclusion Operation				
Total	49	17712.0200		
Between	2	1025.4600	512.7300	1.444 [a] **
Within	47	16686.5600	355.0332	
By Task 3: Provoked Correspondence of Number				
Total	49	17712.0200		
Between	1	4070.7875	4070.7875	14.324 [b] *
Within	48	13461.2325	284.1923	
By Task 4: Addition of Classes				
Total	49	17712.0200		
Between	2	4907.6539	2453.8270	9.007 [a] *
Within	47	12804.3661	272.4333	
By Task 5: Conservation and Measurement of Length				
Total	49	17712.0200		
Between	2	7830.3243	3915.1622	18.622 [a] *
Within	47	9881.6957	210.2488	
By Task 6: Arithmetical Relations of Parts to Whole				
Total	49	17712.0200		
Between	2	1771.6432	885.8216	2.612 [a] **
Within	47	15940.3768	339.1570	

Note. [a] $F(2, 45) = 5.11$ * $p < .01$ and $p < .05$

[b] $F(1, 50) = 7.17$ ** $p > .01$ and $p > .05$

to the ability of the children to perform the selected Piagetian tasks at stages I or III.

Examination of Table 7 indicates a highly significant degree of variation in the means of math concept scores and Task 1, Task 3, Task 4, and Task 5. The F ratio is 44.578 for Task 1; 20.396 for Task 3; 15.769 for Task 4; and 36,617 for Task 5, $p < .01$ and $p < .05$. Even though the scores of children judged as being in the second stage of cognitive development were eliminated, a significant variance exists between the scores' means. In fact, the F ratios for Tasks 1, 4, and 5 exceed the conquerable F ratios in Table 5 for these tasks. The F ratio, 20.396, $p < .01$ for Task 3 is the same as in Table 5 since there were no children in Stage I for either analysis. For Task 6, the F ratio, 4.533, $p < .05$ is significant. The hypothesis is negated for these five Piagetian tasks; a relationship exists between the students' first and third stages of mental development and their scaled math concept scores.

The F of Task 2, 2.822, $p > .01$ is not significant. The means of math concept scores do not differ in variablity for Task 2, thus, this hypothesis may be accepted for the hierarchial classsification and the inclusion operation task.

Table 8 shows a highly significant difference in the means of the total math scores at $p < .01$ and $p < .05$ for: Task 1, $F = 53.383$; task 3, $F = 14.324$; Task 4, $F = 14.445$; and Task 5, $F = 31.032$. For Task 6, $F = 4.254$, $p < .05$, is a significant difference. These F ratios do not fit comfortably in the hypothesized distribution, thus, the hypothesis is rejected for each of these tasks.

The F ratio, 2.025, for Task 2 is less than the critical $F_{.01}(1,23) = 7.88$, and the $F_{.05}(1,23) = 4.28$. The total math level of achievement on

Table 7

Hypothesis 2: The differences in the means of the math concept scores and in the means of the total math scores do not relate directly to the ability of the children to perform the selected Piagetian tasks at stages one or three.

Relationship between Means of Math Concept Scores on the SAT, Form A

Source	df	Sum of Squares	Mean Squares	F Ratio
By Task 1: Seriation, Qualitative Similarity, and Ordinal Correspondence				
Total	32	14264.9697		
Between	1	8413.8569	8413.8569	44.578 *
Within	31	5851.1128	188.7456	
By Task 2: Hierarchial Classification and the Inclusion Operation				
Total	24	7588.6400		
Between	1	829.4400	829.4400	2.822 **
Within	23	6759.2000	293.8783	
By Task 3: Provoked Correspondence of Number				
Total	49	26299.2800		
Between	1	7842.5607	7842.5607	20.396 *
Within	48	18456.7193	384.5150	
By Task 4: Addition of Classes				
Total	43	16324.2500		
Between	1	4455.8839	4455.8839	15.769 *
Within	42	11868.3661	282.5801	
By Task 5: Conservation and Measurement of Length				
Total	45	17172.7174		
Between	1	7800.0217	7800.0217	36.617 *
Within	44	9372.6957	213.0158	
By Task 6: Arithmetical Relations of Parts to Whole				
Total	42	15863.7674		
Between	1	1579.3906	1579.3906	4.533 ***
Within	41	14284.3768	348.3994	

Note. Only students classified as Stage 1 or Stage 3 were used for each task in the comparison of their scaled scores' means.

* $p < .01$ *** $p < .05$

** $p > .01$

Table 8

Hypothesis 2: The differences in the means of the math concept scores and in the means of the total math scores do not relate directly to the ability of the children to perform the selected Piagetian tasks at stages one or three.

Relationship between Means of Total Math Scores on the SAT, Form A

Source	df	Sum of Squares	Mean Squares	F Ratio
By Task 1:	Seriation, Qualitative Similarity, and Ordinal Correspondence			
Total	32	21101.5152		
Between	1	13349.3535	13349.3535	53.383 *
Within	31	7752.1617	250.0697	
By Task 2:	Hierarchial Classification and the Inclusion Operation			
Total	24	11048.5600		
Between	1	894.0100	894.0100	2.025 **
Within	23	10154.5500	441.5022	
By Task 3:	Provoked Correspondence of Number			
Total	49	17712.0200		
Between	1	4070.7875	4070.7875	14.324 *
Within	48	13461.2325	284.1923	
By Task 4:	Addition of Classes			
Total	43	24039.1591		
Between	1	6151.7930	6151.7930	14.445 *
Within	42	17887.3661	425.8897	
By Task 5:	Conservation and Measurement of Length			
Total	45	25608.8696		
Between	1	10591.3913	10591.3913	31.032 *
Within	44	15017.4783	341.3063	
By Task 6:	Arithmetical Relations of Parts to Whole			
Total	42	23151.9070		
Between	1	2176.3602	2176.3602	4.254 ***
Within	41	20975.5468	511.5987	

Note. Only students classified as Stage 1 or Stage 3 were used for each task in the comparison of their scaled scores' means.

* $p < .01$ and $p < .05$ *** $p < .05$

** $p > .01$ and $p > .05$

the Stanford Achievement Test, Form A, appears to be independent of the first and third stages of cognitive development on this task. The hypotesis of independence is accepted.

The second hypothesis, claiming no significant difference in the means of math concept scores and total math scores on the SAT compared to the first and third stages of cognitive development is rejected for five of the six selected tasks. A significant difference exists between the variables for Task 1, Task 3, Task 4, Task 5, and Task 6. The hypothesis is accepted for task 2.

These findings regarding the relationship between scaled math concept or total math scores are not equivocal. The means do differ significantly for Tasks 1, 3, 4, and 5 for the first and second hypotheses and for Task 6 for the second hypothesis. For task 2 the means fit comfortably within the hypothesized chance distribution for the two hypotheses. This is also true for Task 6 for the first hypothesis. A dependent relationship appears to exist between stages of intellectual development and math concept or total math scores' means for the identified Piagetian tasks. This data suggests students in the third stage have higher scores, thus the means are higher than for those measured as "pre-operational" whom tend to have lower math concept or total math scores, which may very well be hampered by their inability to conserve number or to use logico-mathematical reasoning effectively. The relationships for individuals could be assessed by a chart or graphic representation. One needs to keep in mind the scores were influenced by the composition of the select group--40% were ranked in the top ten and 40% in the bottom ten of their class for math ability in daily work.

Third Hypothesis: There is no significant association between children's stages of cognitive development on a particular Piagetian task and their ability to answer specific questions on the math concept section of the

Stanford Math Achievement Test, Form A.

The stages of cognitive development of the children were tabulated and grouped according to their ability to correctly answer specific questions on the math concept test which seemed to be measures of the concept of number related to the Piagetian tasks. These questions were carefully reviewed to select the ones most apt to be measurements of these Piagetian tasks. The relationships were determined separately for first and second grade students from the Stanford Math Achievement Test Primary Level I and Primary Level II, respectively. The math concept questions under consideration in this hypothesis may be located in the 1972 edition of the Stanford Math Achievement Test Booklets, Form A, Primary Level I and Primary Level II. The results are presented in Table 9 and Table 10.

Table 9 indicates that differences found between observed and expected results are not large enough to reject the hypothesis of independence for questions 5, 14, 26, and 27. The results do not show a statistical relationship of dependence at the five per cent level of significance between the specified Piagetian task and the related test item. However, the chi square test reveals significant differences for questions 8, 15, 17, 29, and 32. A positive relationship is recognized between a child's level of cognitive development and his ability to answer the seleded questions. One might assume these SAT items can be utilized in the proper measurement of the abilities of the child to use the selected concrete number tasks.

Examination of Table 10 shows that a statistical relationship is formed at the five per cent level of significance between only questions 29 and 35 and their respective Piagetian task for second grade children. In the remainder of the questions, the observed frequencies are close to those to be expected on the hypothesis of independence and there is no

Table 9

Hypothesis 3: There is no significant association between children's stages of cognitive development on a particular Piagetian task and their ability to answer specific questions on the math concept section of the SAT, Form A.

Piagetian Task	First Grade Math Concept Question	Chi Square (χ^2) Test
Task #1	#5	2.55**
Task #1	#8	8.791*
Task #2	#14	1.55**
Task #2	#15	6.36*
Task #5	#17	4.18[a]*
Task #1	#26	1.77**
Task #3	#27	.33**
Task #1	#29	9.26*
Task #4	#32	12.81*

Note. $p < .05, \chi_c^2 = 5.99$ (2df).

[a] $p .05, \chi_c^2 = 3.84$ (1df).

*$p < .05$.

**$p > .05$.

Table 10

Hypothesis 3: There is no significant association between children's stages of cognitive development on a particular Piagetian task and their ability to answer specific questions on the math concept section of the SAT, Form A.

Second Grade

Piagetian Task	Math Concept Question	Chi Square (χ^2) Test
Task #1	#3	1.326**
Task #4	#7	2.892**
Task #6	#12	.336**
Task #6	#16	2.68**
Task #4	#17	.31**
Task #1	#19	4.402**
Task #4	#22	1.03**
Task #6	#23	1.928**
Task #1	#28	3.39**
Task #1	#29	6.04*
Task #4	#34	.71**
Task #4	#35	6.48*

Note. $p < .05, \chi_c^2 = 5.99$ (2df).

*$p < .05$.

**$p > .05$

evidence of any real association between the specified Piagetian task and these math concept questions. The hypothesis of independence is partially accepted. These results suggest that although the theoretical differences between Piagetian and achievement tests appear to relate to real differences in cognitive measurements of selected items, children's responses on SAT questions, and Piagetian tasks appear to be positively related to each other.

The third hypothesis of independence between stages of mental development on a selected Piagetian task and the related math concept question is partiall y negated. Five questions on the Primary Level I SAT and two items on the Primary Level II SAT show a positive relationship to a selected Piagetian task.

Fourth Hypothesis: There is no significant relationship between children' ability to conserve number as measured by selected Piagetian tasks and their math class achievement ranking as measured by daily classwork.

Examination of Table 11 shows that for each of the Piagetian tasks, except hierarchial classification and the inclusion operation, the fourth hypothesis is rejected and a positive relationship is established between math class ranking and students' stages of intellectual development at the .05 level of significance. These larger five chi square values indicate a real divergence of the experimentally observed from the expected results. Contraiwise, the smaller chi square , 3.766,for the hierarchial classification and inclusion operation task demonstrates the observed results approximate the expected, and thus, the closer agreement between the observed data and the hypothesis being tested.

In general, the students ranking in the top ten of their math class were more apt to be "operational" in their level of logical reasoning than those students ranking in the bottom of their class in math achievement.

Table 11

Hypothesis 4: There is no significant relationship between children's ability to conserve number as measured by selected Piagetian tasks and their math class achievement ranking as measured by daily classwork.

Piagetian Task	Chi Square (χ^2)
TAsk #1: Seriation & Crdinal Corr.	15.824^{a}*
Task #2: Hierarchial Classification	3.766^{a} **
Task #3: Provoked Correspondence	8.534^{b}*
Task #4: Addition of Classes	14.946^{a}*
Task #5 Conservation and Measurement of Length	23.684^{a}*
Task #6 Arithmetical Relation of Parts to Whole	6.88^{a}*

$^{a}p < .05, \chi_c^2 = 5.99$ (df)

$^{b}p < .05, \chi_c^2 = 3.$ (1df)

*$p < .05$

** $p > .05$

This may be due to the fact that in order to deal successfully with grade level work, students need to be "operational" in their thinking regarding Piagetian concept of number tasks, thus, a strong possibility exists they will be in stage three of logical reasoning.

The fourth hypothesis testing the relationship between children's ability to conserve number and their math class achievement ranking is negated for five of the Piagetian tasks. Hierarchial classification and the inclusion operation is the exception.

SUMMARY

In this study, 25 first and 25 second grade children's understanding of logico-mathematical relations were assessed using selected Piagetian tasks to test four hypotheses. The Piagetian tasks included: 1) seriation, qualitative similarity, and ordinal correspondence; 2) hierarchial classification and the inclusion operation; 3) provoked correspondence and conservation of number; 4) addition of classes; 5) conservation and the measurement of length; and 6) arithmetical relations of parts to whole. The children's abilities to conserve number assessed in individual interviews were compared to the means of math concept scores and total math scores of the 1972 Stanford Achievement Test, Form A, using the ANOVA technique. The relationships between these Piagetian tasks and indvidual math concept items on the SAT were assessed using chi square tests. This statistical method was also employed to determine the relationship between children's stages of cognitive development for the Piagtian tasks and their math class achievement ranking as measured by daily classwork.

Hypothesis 1: the differences in the means of the math concept scores and in the means of the total math scores do not relate directly to the ability of the children to perform the selected Piagetian tasks at stages one, two, or three. This hypothesis was negated for Piagetian tasks 1, 3, 4, and 5.; it was accepted for tasks 2 and 6.

Hypothesis 2: the differences in the means of the math concept scores and in the means of the total math scores do not relate directly to the ability of the children to perform the selected Piagetian tasks at stages one or three. This hypothesis was rejected for Piagetian tasks 1,3,4,5,and 6; it was accepted for task 2.

Hypothesis 3: there is no significant association between children's stages of cognitive development on a particular Piagetian task and their

ability to answer specific questions on the math concept section of the Stanford Achievement Test, Form A. The third hypothesis is partially rejected: a relationship is formed between the identified Piagetian task and the math concept questions 8,15,17,29, and 32 on the first grade test and questions 29 and 35 on the second grade SAT, Form A.

Hypothesis 4: there is no significant relationship between children's ability to conserve number as measured by selected Piagetian tasks and their math class achievement ranking as measured by daily classwork. This hypothesis was negated except for the hierarchial classification task. Math class achievement ranking appears to be related to the pupils' ability to deal logically with the remaining five Piagetian tasks.

Basically, if no direct relationship was found between the variables, the hypothesis was accepted; if a direct relationship was determined, the hypothesis was rejected. Either way, useful material is revealed. However, more research is needed to confirm the suggested relationships. Increased sample size might be beneficial.

Teachers might utilize the suggested relationships between children's classroom math ability and their understanding of logico-mathematical relations in designing math and science activities related to pupils' stages of cognitive development instead of endeavoring to teach pre-tested math skills in the same manner to all children. The utilization of a more diagnostic method may be a helpful guide to specific remedial efforts. Even as we acknowledge that the learning process involves planned as well as incidental learning, what is taught should be related to a method of diagnosis of content more descriptive than the numerical score of an achievement test. The achievement test quantifies, whereas, the examination of the meaning of the Piagetian concept provides a response that denotes to the trained teacher wherein the misunderstanding lies.

Future Research

Each new study in cognitive development brings its own discoveries. The results of this study raise questions for future research. These include:

1) How do these six Piagetian taks relate to other standardized achievement tests and to later performance on such tests? How do questions which attempt to measure selected Piagetian concepts compare with an individualized interview method? How can educators use the knowledge demonstarted by these measures to improve instruction? It seems likely that longituidinal studies of individual children may be necessary to possibly obtain these answers.

2) The positive relation exhibited between some Piagetian tasks and certain individual math concept questions on the Stanford Achievement Test, Form A, suggests the possible use of these standardized questions as screening devices for measuring students' cognitive stages of development. The relationship between Piagetian tasks involving time, space, and geometry and individual items on standardized achievement tests should be explored. Some future research should be focused on the development of questions positively related to Piagetian tasks to include in standardized achievement tests. This would enhance their use as evaluative measures of cognitive development and logico-mathematical reasoning abilities.

3) By determining the direct relationships between Piagetian tasks and individual math concept items on achievement or intelligence tests, such as, the Metropolitan Achievement Test, Key Math Test, Iowa Test of Basic Skills, or the California Test of Mental Maturity one may be able to generate a collection of questions based on Piaget's theory of cognitive development to be used in diagnosing individual abilities to use selected concepts for the purpose of prescribing remedial teaching activities. Hopefully, given enough time, money, and creativeness our tests and our teaching methods can be improved to meet this challenge.

The crucial question still exists: Is all this effort worthwhile? I believe so! It would seem self defeating to give tests in which the student merely chooses answers without having the chance to explain his reasoning. It also seems somewhat foolish to continue using evaluative tests lacking a theoretical base when such bases are available. The implied rationality of Piaget's position and his unparalleled conceptualizations provide a dimension to our study of human development and school activities previously missing. Educators have much to gain in applying this research to curricula development, math programs, classroom procedures, and evaluation techniques. Piaget's theories, methodologies, and empirical research will continue to provide an alternative mode of conceptualizing conduct and development and of measuring logico-mathematical behaviors that underguides much of what we teach in schools. Approval or repudiation of a theory is related to its measured applicability, its inductive strength, its logic, and its power to generate new ideas that prove confirmable. Each of these standards is reflected in Jean Piaget's inductive research and his theory of cognitive development. It behooves us to put them to the test.

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